Uncertainty Analysis of Trajectory Tracking for Autonomous Dynamic Soaring

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We quantify the effect of mis-modeled wind conditions on planned and executed energy-neutral trajectories for autonomous dynamic gliding aircraft with a sampling-based probabilistic assessment. Our conclusions indicate the need for future approaches to incorporate model-form error of the wind dynamics into adaptive and/or feedback controllers. While previous works have either analyzed the uncertainty in open-loop controllers developed through either deterministic or stochastic (robust) optimization, our analysis considers the performance of closed-loop (reference-tracking) controllers in the context of both parameter and model-form errors. These controllers use a forward-looking model predictive controller to track a trajectory that was optimized for a deterministic wind profile. The performance of the glider was then simulated using different deterministic wind profiles and gust conditions. The total air-relative energy and glider displacement were then measured for each wind realization to determine the effect of wind field uncertainty on performance. We conclude that the closed-loop controller can efficiently improve the trajectory tracking performance if the deterministic wind profile is not very different from the predicted wind model. However, the closed-loop performance under significant difference in the model form of the wind profile was far from optimal. We find that mis-modelling the underlying deterministic wind profile causes up to 30% loss in energy, significantly more than any effects due to gust.

I. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>parameter determines the wind profile</td>
</tr>
<tr>
<td>$A_L(X)$</td>
<td>linear constraint for X</td>
</tr>
<tr>
<td>a</td>
<td>von Karman constant</td>
</tr>
<tr>
<td>C</td>
<td>cumulative cost function</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>$C_{D_0}$</td>
<td>parasitic drag coefficient</td>
</tr>
<tr>
<td>$C_i$</td>
<td>cost function for a single point</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
</tr>
<tr>
<td>$C_{(x,y,h),(u,v,w)}$</td>
<td>spatial co-coherence decay coefficient for $(u,v,w)$ velocity component along $(x,y,z)$ axis</td>
</tr>
<tr>
<td>$Coh_{(x,y,h),(u,v,w)}$</td>
<td>spatial co-coherence matrix of power density of $(u,v,w)$ components on $(x,y,h)$ axis</td>
</tr>
<tr>
<td>D</td>
<td>drag force</td>
</tr>
<tr>
<td>$Dec_{(u,v,w)}$</td>
<td>spatial co-coherence declination for $(u,v,w)$ velocity component</td>
</tr>
<tr>
<td>$E_f$</td>
<td>the normalized final air-relative energy of the glider</td>
</tr>
<tr>
<td>$E_i$</td>
<td>the normalized initial air-relative energy of the glider</td>
</tr>
<tr>
<td>$E_{max}$</td>
<td>the maximum lift-to-drag ratio of the glider</td>
</tr>
<tr>
<td>$\bar{e_T}$</td>
<td>normalized energy</td>
</tr>
<tr>
<td>f</td>
<td>frequency</td>
</tr>
<tr>
<td>g</td>
<td>gravity constant</td>
</tr>
<tr>
<td>h</td>
<td>height</td>
</tr>
<tr>
<td>$h_{tr}$</td>
<td>height that the wind speed is maximum</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>normalized height</td>
</tr>
<tr>
<td>K</td>
<td>induced drag factor of the glider</td>
</tr>
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</table>

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II. Introduction

Birds are capable of leveraging wind shear [1] and heat thermals [2] to travel long distances without significant energy expense [3,4]. Utilizing these environment features for next generation autonomous aircraft has a great potential to greatly enhance their efficiency and capabilities for a variety of tasks including surveillance and reconnaissance [5,6].

One important challenge to the development of autonomous gliders is the ability to adapt, react, and be robust to an unknown environment. For instance, trajectory generation in typical autonomy applications is performed a-priori to execution. During execution, trajectory tracking controllers are then used to ensure that these plans are followed. However this calculus is potentially complicated in systems such as gliders that are underactuated and potentially underpowered. In other words, because a glider does not carry a powerful propulsion system, it must be able to extract energy from the environment. However, this environment is dynamic and can never be known exactly. These challenges lead to the importance of assessing the effect of uncertainties on performance.

Previous studies of dynamic soaring as an aircraft control problem followed this pattern of separate trajectory planning [7] and path-following [8]. An optimized trajectory under a deterministic wind profile is first established by using numerical optimization of the cycle time [9], thrust or power required [7], or other dynamic soaring performance
criterion determined by the application [10] within a collocation-based framework. In some cases, trajectory following controllers are built to explicitly consider robustness to turbulence [11]. In that work, the high-level controller that generated the reference trajectory considered a fixed underlying wind model while the low-level controller is built to be robust against the gust when tracking the reference trajectory. Notably, one of the primary conclusions was that the lack of robustness of the high-level controller caused performance degradation, despite of robustness of the low-level controller. One notable exception to this separated approach is the work of Patel and Kroo [12], where a genetic algorithm was used to tune PID gains to directly minimize the energy loss of a glider in the presence of Dryden turbulence, without a preliminary trajectory generation.

In virtually all of these works, the trajectory profile is not adapted to changes in the wind. The approaches that consider robustness to turbulence still assume a deterministic and static underlying profile. The analysis provided in this paper shows that such approaches can cause serious barriers to dynamic soaring, even in very simplified settings. We aim to show that effective autonomous dynamic soaring, must not only account for turbulent wind effects but also wide ranging potential wind fields.

Our analysis takes the form of a Monte Carlo uncertainty quantification procedure to simulate the effect of mis-modelled wind conditions on trajectory planning and tracking. Uncertainty analysis of autonomous gliders has been undertaken before under a variety of conditions. For instance the uncertainty of open-loop planners was analyzed by Flanzer and co-workers [13] where it was shown that deterministic open-loop planning results in significant decay of performance when uncertainties are taken into account. They proposed mitigating these effects through a stochastic open-loop planning algorithm computed by solving a robust optimization problem. This approach indeed demonstrated improved performance, and we seek to complement this analysis in two ways. As mentioned, most practical path-planning applications split the trajectory generation and trajectory tracking optimization/control routines, and we explicitly consider the effect of uncertainty even under reference tracking conditions. Furthermore, we also consider model-form errors in addition to parametric uncertainties.

Specifically, we focus on performing uncertainty quantification (UQ) for closed-loop feedback controllers that are computed using forward-looking model predictive control algorithms. We consider three types of uncertainty: parametric uncertainty, model form uncertainty, and stochasticity (gusts). Our results in Section V indicate that model predictive controllers can very effectively deal with parametric and stochastic uncertainty; however, serious performance degradation occurs under model form uncertainty. Overall, our combined contributions are as follows:

1) Uncertainty analysis for open-loop trajectories based on the the model of [7, 9].
2) Uncertainty analysis of closed-loop trajectory-schemes computed using model predictive control, and
3) Uncertainty analysis under parameter, model-form, and stochastic uncertainty.

The rest of this article is structured as follows. In Section III we introduce the wind and vehicle dynamics for which we have obtained preliminary results. There, we also introduce the trajectory generation and tracking schemes. In Section V we detail our preliminary results. In Section VI we discuss the contributions and conclusions. The results of this work are summarized in Figure 1.

Our results indicate that the performance of dynamic soaring using trajectory-following approach is more sensitive to the uncertainty in deterministic wind profile than to gust or parametric uncertainty. Indeed, the optimized trajectory planned for the mis-modelled wind-profile is found to be either not feasible or far from optimal for a different environment. Therefore the robustness of the trajectory tracking based control strategy is largely dependent on the wind field assumption. To improve the performance of dynamic soaring, a possible approach is to avoid global assumption of wind field and make local adaptation for optimal trajectory based on local wind field. Our conclusion is supported by complementary studies in this field [13].

III. Dynamic Models and Trajectory Tracking

In this section, we describe the wind and vehicle dynamics considered in the subsequent analysis. Because we consider both parametric and model form uncertainty in the wind, we first describe several deterministic wind models and their variable parameters. Then we describe the Von Karman turbulence model that we use to consider stochastic forces.
A. Glider dynamics

We model the glider dynamics using a point-mass with six states formulation [9] given by

\[
\begin{align*}
m\dot{V} &= -D - mg \sin \gamma - m\dot{W}_x \cos \gamma \sin \Phi \\
mV \cos \gamma \dot{\Phi} &= L \sin \mu - m\dot{W}_x \cos \Phi \\
mV \gamma &= L \cos \mu - mg \cos \gamma + m\dot{W}_x \sin \gamma \sin \Phi \\
\dot{h} &= V \sin \gamma \\
\dot{x} &= V \cos \gamma \sin \Phi + W_x \\
\dot{y} &= V \cos \gamma \cos \Phi,
\end{align*}
\]

where \( m \) is the glider mass, \( V \) is the speed of the glider, \( \Phi \) is the heading angle, \( \gamma \) is the air-relative flight path angle, \( \mu \) is the glider bank angle, \( L \) is lift force, and \( D \) is the drag force. These forces are given by

\[
\begin{align*}
L &= \frac{1}{2} \rho V^2 SC_L \\
D &= \frac{1}{2} \rho V^2 SC_D \\
C_D &= C_{D_0} + KC_L^2
\end{align*}
\]

where \( C_L \) and \( C_D \) are the lift and drag coefficients, \( C_{D_0} \) is the parasitic drag coefficient, \( K \) is the induced drag factor of the platform and \( E_{\max} \) is the maximum lift-drag ratio of the glider. \( K \) is calculated as

\[
K = \frac{1}{4E_{\max}C_{D_0}}, \quad \textrm{where} \quad E_{\max} = \left( \frac{C_L}{C_{D_0} + KC_L^2} \right)_{\max}.
\]

Normalization of the equations of motion and \( \dot{W}_x \)

To ensure the numerical simulation of the flight dynamics general for gliders with different mass and wing area, the dynamics of the glider is normalized according to [7] to obtain a normalized system with two control inputs (\( C_L \) and bank angle \( \mu \))

\[
x' = f(x, u), \quad \textrm{where} \quad x = [\tilde{V} \; \Phi \; \gamma \; \tilde{h} \; \tilde{x} \; \tilde{y}]^T \; \textrm{and} \; u = [C_L \; \mu].
\]

We define the time normalization constant is \( \beta \) and defined by

\[
\beta = \frac{w_{ref}}{\dot{h}_{ref}}.
\]
so that the normalized time is
\[ \tau = \beta t. \]  
(5)

The normalized displacements and air-density are
\[ (\bar{x}, \bar{y}, \bar{h}) = \frac{(x, y, h) \beta^2}{g}, \]  
(6)
\[ \bar{\rho} = \frac{\rho g^2}{2(mg/S)\beta^2}. \]  
(7)
respectively. Furthermore, the normalized velocity is
\[ \bar{V} = \frac{V \beta}{g}. \]  
(8)
The normalized time derivatives become
\[ (\cdot)' = \frac{d(\cdot)}{d\tau} = \frac{1}{\beta} \frac{d(\cdot)}{dt}, \]  
(9)
and so the the normalized rate of change of wind velocity is derived using chain rule:
\[ \bar{W}_x' = \frac{\partial \bar{W}_x}{\partial \bar{x}} \bar{x}' + \frac{\partial \bar{W}_x}{\partial \bar{y}} \bar{y}' + \frac{\partial \bar{W}_x}{\partial \bar{h}} \bar{h}'. \]  
(10)
The normalized wind gradient is
\[ \frac{\partial \bar{W}_x}{\partial (\bar{x}, \bar{y}, \bar{h})} = \frac{g}{\beta^2} \frac{\partial \bar{W}_x}{\partial (x, y, h)}. \]  
(11)
Together, the normalized equations of motion become
\[ \bar{V}' = -\bar{\rho} \bar{V}^2 (C_{D_0} + KC_L^2) - \sin \gamma - \bar{W}_x' \cos \gamma \sin \Phi \]  
(12)
\[ \Phi' = \bar{\rho} \bar{V} C_L \sin \mu \cos \gamma - \frac{\bar{W}_x' \cos \Phi}{\bar{V} \cos \gamma} \]  
(13)
\[ \gamma' = \bar{\rho} \bar{V} C_L \cos \mu - \frac{\cos \gamma}{\bar{V}} + \frac{\bar{W}_x' \sin \gamma \sin \Phi}{\bar{V}} \]  
(14)
\[ \bar{h}' = \bar{V} \sin \gamma \]  
(15)
\[ \bar{x}' = \bar{V} \cos \gamma \sin \Phi + \bar{W}_x \]  
(16)
\[ \bar{y}' = \bar{V} \cos \gamma \cos \Phi. \]  
(17)
From these normalized quantities we obtain the total normalized air-relative energy and air-relative energy rates as
\[ \bar{\epsilon}_T = \bar{h} + \frac{\bar{V}^2}{2}, \]  
\[ \bar{\epsilon}_T' = -\bar{\rho} \bar{V}^3 (C_{D_0} + KC_L^2) - \bar{W}_x \bar{V} \sin \gamma \cos \gamma \sin \Phi. \]

We set \( \bar{\rho} = 60 \text{ ft}^{-2} \) as constant throughout the following sections and the rest of the parameters: \( \beta, h_{tr}, E_{max} \) and \( C_{D_0} \) are sampled around their nominal values as described in Table 2.

B. Wind Models

Next we describe the three wind-profiles and the single turbulence model that we use for analysis. The total horizontal wind velocity with respect to ground \( W_x \) is the sum of deterministic wind velocity \( \bar{W}_x \) given by the wind profiles \( \bar{W}_x (\bar{h}) \) and the horizontal gust component \( u_{gust} \)
\[ W_x = \bar{W}_x + u_{gust}. \]  
(18)
where

\[ f_{\text{max}} = f_s/2 \]

is equal to half of the sampling frequency \( f_s \) in Table 1, and \( f_{\text{min}} \) give the finest resolution that we considered. The time duration for the gust generation is significantly smaller than the duration of flight.

The gust is simulated as an ergodic stationary stochastic process as described by Shinozuka and Deodatis [18]. The gust velocity is approximated by a cosine series with minimum frequency \( f_{\text{min}} \) and maximum frequency \( f_{\text{max}} = f_s/2 \) is equal to half of the sampling frequency \( f_s \). The discrete frequencies used have an increment of \( \Delta f = f_{\text{min}} \). At each frequency, the gust velocity is spatially correlated with the co-coherence decaying exponentially with distance as formulated by Davenport [19]. The wind co-coherence matrix is defined according to

\[ \text{Coh}_u(i, j) = f \left( \frac{u(i) + u(j)}{2} \right)^{-1} e^{-\sqrt{\text{Dec}_{xu}(i, j)^2 + \text{Dec}_{yu}(i, j)^2 + \text{Dec}_{zu}(i, j)^2}}, \]

where the spatial decay coefficient matrix is defined as

\[ \text{Dec}_{xu}(i, j) = C_{xu}(x(i) - x(j)), \]

and \( \text{Dec}_{xu}(i, j) = 0 \) for 2D gust in \( x - h \) plane. Here, \( x(i) \) and \( x(j) \) denote the x-coordinate of node \( i \) and \( j \), and \( \text{Dec}_{xu}(i, j) \) denotes the decay coefficient of wind component \( u \) along x-direction between these two nodes. The subscripts \( u, v, w \) denote the longitudinal, lateral and vertical velocity component with respect to the local velocity of the deterministic wind profile. The three components are independent of each other, and therefore \( v, w \) components are not needed in our one-directional analysis. The spatial decay coefficients \( C_{xu} \) and \( C_{hu} \) are set large enough so that the wind co-coherence be well conditioned.

The spectral power density (PSD) of the stochastic process is given by Von Karman turbulence model, a semi-empirical form for continuous gust,

\[ \Psi_u(f, h) = \sigma_u^2 W(h) \left[ 1 + \left( 2 \pi a f \frac{L_u}{W(h)} \right)^2 \right]^{5/6}, \]

where Von Karman constant \( a = 1.339 \).
for the longitudinal gust component and
\[
\Psi_{v,w}(f,h) = \sigma_{v,w}^2 \frac{2L_{a,v,w}}{W(h)} \left( \frac{1 + \frac{8}{3}(2\pi a f L_{a,v,w} W(h))^2}{[1 + (2\pi a f L_{a,v,w} W(h))^2]^{1/6}} \right),
\]
where Von Karman constant \( a = 1.339 \) (25)

for the lateral components at each frequency \( f \) at the altitude \( h \). The deterministic wind velocity with respect to ground at the altitude \( h \) is denoted as \( W(h) \) and the turbulence length scales which comes from empirical measurements are denoted as \( L_{a,v,w} \). Finally, the cross-spectral density is given by
\[
S_{uu} = \sqrt{\Psi_u \Psi_u^T} \text{Coh}_{uu}, \quad \text{(26)}
\]
and the simulated gust velocity by
\[
u_{Gust} = \sum_{i=1}^{n_f} \sqrt{2\Delta f \delta f} |G_i| \cos(\omega_i t_i + \theta_i), \text{ where } \omega_i = 2\pi f_i \text{ and } G = \text{chol}(S_{uu}). \quad \text{(27)}
\]
At each frequency, a random phase angle \( \theta_i \) is generated in \([0, 2\pi]\) as independent uniform random variable. The gust velocity and its rate of change are normalized in the same way as deterministic wind velocity and rate of change.

A summary of the parameters for the turbulence generation is listed in Table 1.

<table>
<thead>
<tr>
<th>Value</th>
<th>( \text{Parameter} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.6</td>
<td>( \sigma_{u}[ft/s] )</td>
</tr>
<tr>
<td>32.5</td>
<td>( \sigma_{w}[ft/s] )</td>
</tr>
<tr>
<td>1000</td>
<td>( h_r[ft] )</td>
</tr>
<tr>
<td>190.5</td>
<td>( L_u[ft] )</td>
</tr>
<tr>
<td>20</td>
<td>( C_{uu} )</td>
</tr>
<tr>
<td>20</td>
<td>( C_{hu} )</td>
</tr>
<tr>
<td>100</td>
<td>( N_x )</td>
</tr>
<tr>
<td>100</td>
<td>( N_z )</td>
</tr>
<tr>
<td>2</td>
<td>( f_s[Hz] )</td>
</tr>
<tr>
<td>5</td>
<td>( T_{max}[s] )</td>
</tr>
</tbody>
</table>

Table 1  Turbulence model parameters

IV. Trajectory generation and tracking

In this section we describe the trajectory generation and tracking approaches.

A. Open-loop trajectory generation

Open loop trajectory generation seeks a traveling pattern, where the goal of the glider is to travel only in the \( y \) direction with respect to the initial location. The most common approach for open-loop trajectory generation in this context is based on a collocation method, where the time is discretized and an optimal control is sought for every time period \([9, 10, 14]\). We follow this most common procedure, though alternatives exist [20], because our goal is to assess the performance of standard techniques in the presence of uncertainty.

Our approach follows directly that of [9]. We compute the optimal trajectory by discretizing the state and control into \( N = 32 \) elements in a period, each one separated by a distance of \( \Delta t = \tau_f/N \), where \( \tau_f \) is the optimized trajectory completion time. Let \( x_k^i \) represents the \( i \)th state during the \( k \)th time step for \( i = 1, \ldots, 6 \) and let \( u_k^i \) represents the control input during the \( k \)th time step for \( j = 1, 2 \). All of these variables can be grouped into a large vector
\[
X = [x_1^1, \ldots, x_1^N, \ldots, x_N^1, \ldots, x_N^6, u_1^1, \ldots, u_N^1, u_1^2, \ldots, u_N^2, \tau_f], \text{ (28)}
\]
over which the optimizer searches for an optimal trajectory. Following [9] we also add non-linear equality constraints \( R(X) = 0 \) for those \( 8N + 1 \) variables to assure the trajectory follows the dynamic model [3]. Furthermore, we specify a constraint that the final trajectory values for all but the \( y \) position should be equal to their initial values; this constraint will enforce that the glider travels only in the \( y \) direction and is denoted by \( A_L(X) = 0 \).

Together, our optimization problem seeks to minimize the period subject to these constraints:
\[
\min_{X} \tau_f \quad \text{subject to} \quad L \leq X \leq U, \quad A_L(X) = 0, \quad R(X) = 0, \quad \text{(29)}
\]
where, \( L \) and \( U \) are the lower and upper bound for each state.
B. Reference trajectory tracking

A glider will virtually never have complete knowledge of the environment. As a result, the trajectories described above are used as inputs to reference tracking controllers for increasing robustness. The hope is that reference-tracking controllers can compensate for any mis-modeled effects to keep the aircraft on track.

Several approaches for reference tracking can be considered, for instance Bird and Langelaan [8] construct a reference tracking using a linearized feedback controller. Here, we consider a model-predictive approach, where a feedback controller is obtained at each timestep by optimizing over a future horizon of \( N_{\text{forward}} \) timesteps. For each timestep we specify a quadratic penalty on the state deviating from the reference. This penalty is

\[
C(x(t), u(t)) = ||x(t) - x_{\text{ref}}(t)||^2_Q,
\]

where \( x_{\text{ref}} \) and \( u_{\text{ref}} \) are the reference state variables and control variables obtained from the open-loop trajectory and \( Q = \text{diag}[0 \ 0 \ 0 \ 1 \ 1] \), denotes a weight on the penalty of deviating from a reference trajectory.

To formulate a forward-looking optimization problem we solve the following optimization problem

\[
\begin{align*}
[\mathbf{u}^*(t), \ldots, \mathbf{u}^*(t + N_{\text{forward}}\Delta t)] = & \arg\min_{(\mathbf{u}(t+i\Delta t))_{i=0}^{N_{\text{forward}}}} \sum_{i=1}^{N_{\text{forward}}} C(x(t+i\Delta t), u(t+i\Delta t)) \\
\text{subject to } \Delta t. 
\end{align*}
\]

In this case \( \Delta t \) is the dynamics increment used to constrain the trajectory generation as described in the previous section. The differential constraints are satisfied through a forward Euler simulation of the dynamics with a time step of \( (\Delta t)/32 \), where the controls are linearly interpolated within each \( \Delta t \) timestep. This optimization is performed using the Sequential Quadratic Programming (SQP) algorithm provided within \texttt{fmincon} in MATLAB.

V. Results

We now provide the setting and results of our uncertainty propagation analysis. All uncertainty propagation is performed via Monte Carlo sampling using 10,000 samples for each configuration. Furthermore, we consider simulations that lose more than 30\% of their energy for their energy to have “failed” at soaring. We also define a path to be feasible that lose more than 30\% of their energy for their energy to have “failed” at soaring. We also define a path to be feasible that lose more than 30\% of their energy for their energy to have “failed” at soaring.

To the power-curve wind model given by Equation (20) and the logarithmic wind model given by Equation (21) we consider uncertainty with respect to four parameters: \( \beta, P_1, C_{D_0}, E_{\text{max}} \). All of these parameters are uniformly distributed around a nominal value. The number of such samples for each case is indicated, and these samples are discarded from the figures.

For the wind model given in Equation (19) we consider uncertainty with respect to five parameters: \( \beta, h_{tr}, C_{D_0}, E_{\text{max}} \), and \( A \). We endow these parameters with a uniform distribution around a nominal value. The bounds of the uniform distribution are chosen to be \( \pm 10\% \) deviation from the nominal value for \( \beta, h_{tr}, C_{D_0} \) and \( E_{\text{max}} \). The parameter \( A \), defining the shape of the wind profile, we give a larger variation that is \( \pm 50\% \) deviation from nominal value. All of the resulting parameter uncertainties are summarized in Tables 2 and 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \beta )</th>
<th>( A )</th>
<th>( h_{tr} )</th>
<th>( C_{D_0} )</th>
<th>( E_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution</td>
<td>( \mathcal{U}[0.027207, 0.033253] )</td>
<td>( \mathcal{U}[0.5, 1.5] )</td>
<td>( \mathcal{U}[1800, 2200] )</td>
<td>( \mathcal{U}[0.009, 0.011] )</td>
<td>( \mathcal{U}[22.5, 27.5] )</td>
</tr>
</tbody>
</table>

Table 2 Uncertain parameters and their ranges for the wind model of (9) given by (19).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution</td>
<td>( \mathcal{U}[2.16 \times 10^{-8}, 2.64 \times 10^{-8}] )</td>
<td>( \mathcal{U}[7.65, 9.35] )</td>
</tr>
</tbody>
</table>

Table 3 Additional uncertain parameters for the power (20) and logarithmic wind models (21).

We consider a number of test cases to analyze. The reference trajectory is developed in Section V.A. We then analyze the performance of the open-loop and the closed-loop controllers using the nominal wind model given in Equation (19).
The open-loop results are provided in Section V.B and the closed-loop results are provided in Section V.C. Next, we consider model-form uncertainty by repeating these experiments while using a different wind model for simulation. In particular Section V.D provides results for tracking the reference trajectory while flying under the power- and logarithmic-wind models. Finally, we consider both model form uncertainty and a stochastic model by introducing turbulence. These results are provided in Section V.E.

Our main analysis consists of marginal distributions of the energy ratio and distance of deviation from the target path. Perfect soaring would have an energy ratio of one and a deviation distance of zero. These results are shown in Figures 5, 7, 8, and 9. Each of these plots has a set of intersecting red lines. These red lines have the same length in all plots and are used as a scale reference. They are centered around the performance of a closed-loop trajectory under known wind parameters and no gust.

A. Reference trajectory

We first present the nominal trajectory computed using open-loop trajectory optimization of Equation (29). The nominal trajectory with least period is shown in Figure 2. In this plot we show the resulting trajectory found by optimization on the 32 collocation points, and a corresponding trajectory obtained by interpolating the resulting controller in continuous time and integrating with an adaptive Runge-Kutta-45 scheme. We see that while there is some deviation between the computed optimal trajectory and the more accurate time integrated trajectory, this deviation is minimal.

Figure 3 shows the nominal control values. This result validates the starting point provided by the previous work of [7]. Figure 4 shows the total normalized energy where it is seen that the initial and final energies are virtually identical, indicating an energy neutral trajectory is achieved.

B. Uncertainty quantification of open-loop trajectory tracking

Using 10000 MC samples, we assessed the performance of the open-loop controls shown in Figure 3 with respect to various wind model parameters. These results are shown in Figure 5.

The histograms illustrate that even with a little uncertainty, the trajectories deviate significantly from optimality. Furthermore, a large portion of the feasible trajectories still lose energy, and for others the glider moves a significant extent in non-intended directions. Indeed over 30% of the resulting trajectories result in infeasible paths. To illustrate these types of trajectories we show examples of failed, suboptimal, and good trajectories in Figures 6a, 6b, and 6c.
Fig. 3 Nominal controls for reference trajectory. The dash curve is the initial optimization point and the solid curve represents the optimized results.

Fig. 4 The total normalized energy of travelling pattern of a glider after optimization. The total normalized energy in y-axis, time (s) in x-axis. The dash curve is the initial guess. The solid curve represents the optimized results. Note that the initial and final energy are equal.

C. Uncertainty quantification with close-loop trajectory tracking

Next we consider the performance closed-loop trajectory tracking. Using the same 10000 MC samples, the resulting initial-final air-relative energy ratio and $x-$position deviations are shown in Figure 7. No failures occur with the closed-loop trajectory tracking. Furthermore, the energy gain after a period stay very close to 1, which means the glider can fly for more periods for a longer period of time. Additionally, the travelled $x$ distance shifts less than for the open-loop trajectories, indicating a better realization of the travelling pattern.

D. Uncertainty quantification with close-loop trajectory tracking under model form uncertainty

Next, we see if these reference tracking results carry over to the power and logarithmic wind models. The one-dimensional marginals and two-dimensional joint distributions of initial-final air-relative energy ratio and $x-$direction travel for the power-like model as well as their joint distribution are shown in Figure 8a for the power-like model, and in Figure 8b for the logarithm model. Here, the tracking method suffers, and results in a significant loss of energy every period. Furthermore, the drift in the $x$-direction displacement becomes very large, causing the targeted traveling behavior to fail.
Fig. 5 Histogram of energy change and final position of the glider under 10000 random samples of wind conditions using the open-loop trajectory. The left most bar in each plot denotes that a large portion of the trajectories resulted in failures. The red line indicates the performance of the open-loop trajectory as simulated using an adaptive integrator.

Fig. 6 Examples of failed, suboptimal, and good trajectories realized through Monte Carlo assessment of the performance of the open-loop policies for the wind model given by Equation (19). Blue: input optimal trajectory. Black: actual trajectory. Red: extension of the new trajectory according to prediction.

E. Uncertainty quantification of close-loop tracking under turbulence and mis-modeled wind

We also considered an assessment of the effect of a turbulence model on control performance. This extension to the existing results was made by using the same model predictive controller as for the deterministic case, however, we added stochasticity of the wind in the simulation. Three settings were considered. The first was a case where the underlying deterministic wind profile is fixed and only gusts are added, and the second was where Monte Carlo sampling occurs over both the wind profile and the stochastic realizations. The final case has the same uncertainty as the second one, however the model predictive controller does not know the true underlying wind profile, and instead plans each step under the nominal parameters. The results for these three cases are shown in Figure 9.

Figure 9a shows that in the case where the underlying wind model is fixed and only stochastic gust is considered, the controller is robust toward gust and the optimal trajectory is still feasible because the ΔE of successful trajectory following cases are close to energy neutral and the drifts in x-direction is close to zero.

Figure 9b shows that in the case where both parametric and stochastic uncertainty is considered, the controller is still able to perform adequate tracking of the optimal trajectory. However, there are significantly more failed trajectories than in the purely stochastic case. Recall that in this case, the forward-looking controller knows the underlying nominal model form and parameters for each simulation.

The final case is shown in Figure 9c; here the controller is assumed to know the underlying model form given by Equation (19); however, it does not know the specific set of parameters and plans for the future using the nominal
Fig. 7 Joint density plot of $\Delta X$ and $E_f/E_i$ for the closed-loop trajectory tracking around the nominal trajectory with only parameter uncertainty.

Fig. 8 Joint density plot of $\Delta X$ and $E_f/E_i$ for the closed-loop trajectory tracking under model-form error.

parameters. Here again, we see that the controller is able to adequately track the trajectory with 534 failures. Compared to the case where there is model-form error; this performance is adequate.

VI. Conclusions

In this work we have analyzed the traditional trajectory generation followed by tracking approach for dynamic soaring aircraft from an uncertainty quantification viewpoint. Our results indicate that this traditional approach only seems to be robust and suitable (under the tested conditions) when the model used to generate the trajectory is from
Fig. 9 Joint density plot of $\Delta X$ and $E_f / E_i$ for close-loop tracking under three different conditions that consider stochastic uncertainty due to gust. Parameter uncertainty is considered in the second two panels. The middle panel uses a controller that knows the form for each simulation, while the right-most panel uses parameters of the fixed nominal wind form.

The same family or class as that used during actual operations. In these cases, even under parameter variability and stochasticity (gusts) a model predictive control approach was able to perform efficient soaring.

Nevertheless, for the more realistic case where the actual wind follows a different profile than that under which reference trajectory were generated, the controller is unable to compensate for this model mis-specification. Moreover, energy losses were found to be almost 30% and significant drifting was observed. These analysis were in the context of a simplified wind model setting where the differences between the nominal and tested wind models are fairly reasonable. We expect that this problem is only exacerbated in more realistic 3D wind configurations.

In the closed-loop tracking cases where stochastic forcing on model predictive approach is included through Von Karman turbulence we observed that the turbulence doesn’t affect the correlation between the change of energy and deviation of trajectory in the $x$-direction at the same level as the deterministic wind profile. In other words, the controller is demonstrated to be robust toward unexpected gust. To summarize, the underlying model form uncertainty of the wind-profile has a more significant impact on the performance of dynamic soaring than either parametric uncertainty or stochasticity.

These results indicate that future autonomous systems will have to persistently learn and re-plan as conditions shift. Robustness will likely require moving away from the traditional trajectory planning and tracking paradigm.

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References


